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DETERMINATION OF STRESS INTENSITY FACTORS DURING CRACK ARREST IN DUPLEX SPECIMENS

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Abstract—The objective of this paper is to indicate that in the photoelastic determination of stress intensity factors (K) in duplex specimens and when the crack meets the interface the particular value of the elastic stress singularity, different in general from the value 0.5, should be taken into account. It is shown that the thus introduced correction in the determination of K by using the well-known formula based on the value of stress singularity equal to 0.5 is of great significance when the elastic moduli of the two materials differ substantially, while it can be omitted when these elastic moduli are close to each other.

Duplex specimens used in fracture arrest toughness measurements present many advantages over the well known monolithic specimens[1]. These specimens were used by many investigators in studying crack propagation and arrest phenomena[1–4]. A duplex specimen consists of a starter section made of a brittle material and an arrest section made of a tougher material (Fig. 1a). The two sections are bonded together along a straight line so that continuity of stresses and displacements are to be ensured and an initial crack is made in the starter section perpendicular to the common interface. The bond is made either by casting the one section to the other or by fastening the two sections along a joint line. The fastening of the two materials if they are plastics is made by using an appropriate glue[3, 4], while for metallic specimens it is achieved by electron beam welding. Effort is made for the thickness of the glue or the welding to be as small as possible.

A series of tests in duplex specimens aiming to determine the velocities and the stress intensity factors during crack propagation took place at the University of Maryland[3, 4]. The method of dynamic photoelasticity in conjunction with a Cranz–Schardin high speed camera was used for the determination of K_I at the various stages of crack propagation. It was found that the crack advances with an exactly straight front and therefore only the K_I opening mode stress intensity factor exists during crack propagation. A typical diagram of the variation of K_I vs time t as it was found in Refs. [3, 4] (Fig. 6 of Ref. [4]) is presented in Fig. 2. For the determination of K_I data from the isochromatic fringe loops in the vicinity of the crack tip were taken. The value of K_I was determined from the relation[5]:

$$K_I = \frac{Nf(2\pi r_m)^{1/2}}{h \sin\theta_m}\left[1+\left(\frac{2}{3\tan\theta_m}\right)^2\right]^{-1/2}\left(1+\frac{2\tan\frac{3\theta_m}{2}}{3\tan\theta_m}\right) \qquad (1)$$

where N is the fringe order at the point with polar coordinates (r_m, θ_m), f is the photoelastic fringe value and h the thickness of the specimen.

In the derivation of the formula (1) the singular stress field solution of a crack in an isotropic medium in conjunction with a constant normal stress parallel to the crack line was taken into account. This solution presents an inverse square root singularity.

We will now pay attention to the behavior of the $K_I - t$ curve during the arrest time with the crack tip lying in the joint phase. The experimental situation of a crack perpendicular to a bimaterial interface can be simulated by the corresponding ideal model of a crack terminating perpendicularly to a planar interface between two different isotropic materials. This ideal model simulates exactly the real specimen for the first of the above ways of preparing the duplex specimen according to which the one section is cast to the other. For the second way in which a different material is used to fasten the two sections the ideal model is closer to the real specimen as the thickness of the fastening material is smaller. However, for appreciable thicknesses of the fastening material the ideal model fails to simulate the actual duplex

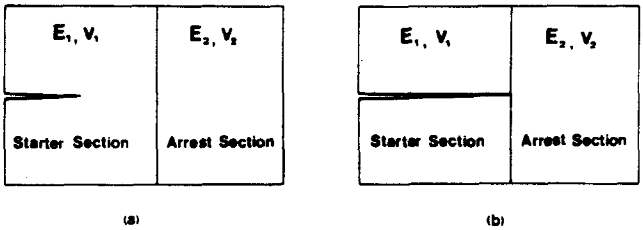

Fig. 1. Duplex specimen with an initial crack (a) and with the crack approaching the interface (b).

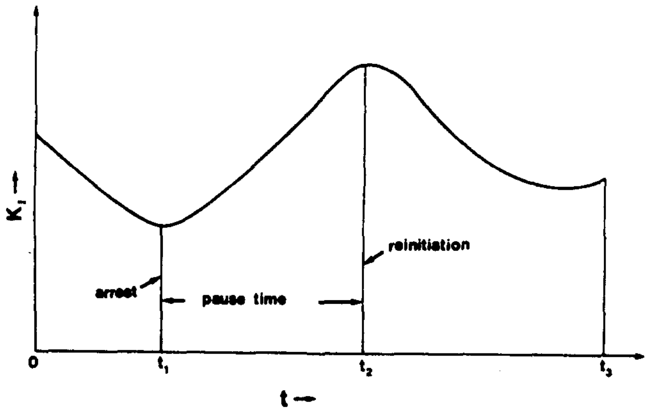

Fig. 2. Variation of K_I stress intensity factor with time t during crack propagation in a duplex specimen.

specimen. In such cases the fastening material should be taken into account as the third phase of the specimen.

For such case the crack tip stress singularity is no longer equal to 0.5. This problem was first considered by Zak and Williams[6] who found that the value of the stress singularity g is strongly dependent on the elastic properties defined by the moduli of elasticity $E_{1,2}$ and the Poisson's ratios $\nu_{1,2}$ of the two constituent media. Furthermore, they found that g is always real for any material combination of the two media. Therefore, the singular behavior of the stresses in the vicinity of the crack tip will be of the form r^{-g}. The value of g depending on the values of $E_{1,2}$ and $\nu_{1,2}$ may vary from $g = 0$ to $g = 1$. Contour lines of $(1-g)$ were given by Bogy[7] in the Dundurs parallelograms for all material combinations of the two media.

From the above discussion and with the value of g not equal to 0.5 it is concluded that the formula (1) for evaluating the value of K_I is no longer valid. If now it is assumed that K_I retains its meaning as the factor who defines the magnitude of the stress field in the vicinity of the crack tip and that the singular stresses are given by the well known formulas with the value of the elastic stress singularity being equal to g, we conclude that for the determination of K_I the value of $r^{1/2}$ must be replaced by r^g in formula (1). This formula contains some degree of approximation since the angular stress distribution is no longer the same as in the case of a crack tip in an isotropic medium. It can however be observed that the influence of the power of singularity in evaluating the isochromatic fringe loops in the vicinity of the crack tip is stronger than the influence of the angular stress distribution and therefore the correction of formula (1) by introducing the particular value of stress singularity different than (1/2) gives a good approximation.

In order to get an idea of the correction introduced by using the correct form of formula (1) for evaluating K_I let us consider an example. In the duplex specimen of Fig. 1 the starter section is made of an epoxy resin with $E_1 = 4.5 \times 10^5$ psi, $\nu_1 = 0.35$ and the arrest section is made of aluminum with $E_2 = 10^7$ psi, $\nu_2 = 0.30$. For such case the order of the elastic stress singularity as it can be deduced from Ref. [7] is equal to $g = 0.33$. In the K_I evaluation let us take three points with $r_m = 0.001$, 0.01 and 0.1 and an arbitrary value of θ_m. The closer to the crack tip the point selected the more accurate is the validity of formula (1). For these three points the values of $r^{1/2}$ are equal to: 0.032, 0.100, 0.316, while the values of $r^{0.33}$ are equal to: 0.102, 0.219, 0.468.

It is therefore concluded that using formula (1) the values of K_I are always underestimated by factors of the order 3, 2, 1.5 depending on the value of r.

In the above example due to the large mismatch of the elastic moduli of the two materials the order of the stress singularity was substantially different than the value 0.50 and therefore great correction factors should be introduced. For the case however, when the ratio of these elastic moduli is small the correction introduced is of minor significance. Thus, for the case of the duplex specimens used in Ref. [4] it is $(E_1/E_2) = 1.2$. If now we accept that the model of Fig. 1 simulates exactly the actual specimens then $g = 0.49$ and for $r_m = 0.001$, 0.01 and 0.1 the correction factors are 0.94, 0.96, 0.98 respectively. We observe that in this case the correction in the calculation of the stress intensity factor is small.

From the above developments it is concluded that the value of the elastic stress singularity, different in general from the value of 0.5, should be taken into account in the evaluation of K_I as the crack meets the interface of the duplex specimen. The thus introduced correction in the determination of K_I by using formula (1) based on the value of stress singularity equal to 0.5 is of great significance when the elastic moduli of the two materials of the duplex specimen differ substantially. However, when the ratio of the elastic moduli approaches the unity and the data on the isochromatic pattern are not taken too near to the crack tip the correction factor is small and can be omitted.

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